Lie Group time integration of a nonlinear geometrically exact Cosserat rod model

Stefan Hante (Martin Luther University Halle–Wittenberg), Martin Arnold

Guided by the analysis in [1], we present a nonlinear geometrically exact Cosserat rod model, that can be obtained by applying a Lie group variational principle in time and space for linear, viscoelastic material behaviour. The rotational components are represented using unit quaternions $p \in \mathbb{S}^3$ (Euler parameters). By adding constraints, a second order partial differential algebraic equation with solution in the Lie group $\mathbb{R}^3 \times \mathbb{S}^3$ is obtained. Following the method of lines the equation is semidiscretized in space using a staggered grid (see [2]), yielding a second order differential algebraic equation of index three with solution in the Lie group $(\mathbb{R}^3 \times \mathbb{S}^3)^n \times \mathbb{R}^3$.

A general purpose Lie group generalized-α solver is then applied in order to solve the index-three DAE either directly or using a stabilized index-two formulation. As a test scenario we use the 'flying spaghetti' benchmark problem, in which forces and moments are applied to an unhitched straight rod, resulting in three-dimensional rotations.

References

[1] Holger Lang and Joachim Linn. Lagrangian field theory in space-time for geometrically exact Cosserat rods. *Ber. ITWM Kaiserslautern, 150*, 2009.

[2] Holger Lang, Joachim Linn, and Martin Arnold. Multi-body dynamics simulation of geometrically exact Cosserat rods. *Multibody System Dynamics, 25*, 285–312, 2011.