A convergence analysis for the shift-and-invert Krylov method

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Time integration methods for stiff systems of ordinary differential equations often involve the action of a matrix function $f(A)$ on a vector b. The matrix A typically arises from a spatial discretization of a partial differential equation and has a huge field-of-values lying somewhere in the left complex half-plane. Refining the discretization, the norm of A becomes very large. Therefore, the efficient and reliable approximation of $f(A)b$ with a convergence rate independent of $\|A\|$ is a current topic of interest and research.
Recent advances have shown that rational Krylov subspace methods have a great advantage over standard Krylov subspace methods in this case. We thus approximate $f(A)b$ in the shift-and-invert Krylov subspace

$$\mathrm{span}\{b, (\gamma I - A)^{-1}b, (\gamma I - A)^{-2}b, \ldots, (\gamma I - A)^{-(m-1)}b\}\,, \quad \gamma > 0\,.$$

By transforming the left complex half-plane to the unit disk, we obtain convergence results that depend on the smoothness of a transformed function on the boundary of this disk. In particular, we establish sublinear error bounds for the matrix φ-functions being of central importance in exponential integrators. A remarkable aspect of our analysis is the independence of the error from the norm of the considered discretization matrix.
Moreover, we discuss suitable choices for the shift γ in the shift-and-invert Krylov subspace and illustrate our results by several numerical experiments.