Error Control in Solving Differential Algebraic Equations of High Order

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Consider a linear system of ordinary differential equations

$$A_k(t)x^{(k)}(t) + A_{k-1}(t)x^{(k-1)}(t) + \ldots + A_0(t)x(t) = f(t),\ t \in T := [0, 1] \quad (1)$$

where $A_i(t)$ are $(n \times n)$-matrices, $i = \overline{1,k}$, $x(t)$ and $f(t)$ are the desired and the given vector-functions, correspondingly, with the initial data

$$x^{(j)}(0) = a_j,\ j = \overline{0, k-1}, \quad (2)$$

It is assumed that the input data is smooth enough and the following condition holds

$$\det A_k(t) = 0\ \forall t \in T. \quad (3)$$

In this talk, we introduce the notion of an index for systems (1) with the condition (3). Then, in terms of matrix polynomials, we obtain a criterion for the index of (1) not to exceed k. Provided that the criterion is fulfilled, we consider a difference scheme for solving (1),(2) and demonstrate that for the perturbation

$$\tilde{f}(t) = f(t) + \mu(t), \|\mu(t)\|_{\mathbf{C}} \leq \delta,$$

where δ is a small real number, the function of error depends both on the integration step and the level of perturbation.

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