Volume Preservation by Runge-Kutta Methods

Philipp Bader (La Trobe University), D. I. McLaren, M. Webb, GRW Quispel

It is a classical theorem of Liouville that Hamiltonian systems preserve volume in phase space. Any symplectic Runge-Kutta method will respect this property for such systems, but it has been shown that no B-Series method can be volume preserving for all volume preserving vector fields (BIT 47 (2007) 351-378 and IMA J. Numer. Anal. 27 (2007) 381-405). We show that despite this result, symplectic Runge-Kutta methods can be volume preserving for a much larger class of vector fields than Hamiltonian systems, and discuss how some Runge-Kutta methods can preserve a modified measure exactly.