A doubling-splitting approach for the fractional heat equation

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Fractional order differential equations, as generalizations of classical order differential equations, are increasingly used in model problems in fluid flow, in finance and other areas of application, such as advection-dispersion models from hydrology. In this presentation I discuss the space-fractional heat equation:

$$u_t = D_{\mathcal{C}}^{\alpha} u, \ 1 < \alpha \leq 2, \tag{1}$$

in which the fractional derivative $D_{\mathcal{C}}^{\alpha}$ is defined 'in the sense of Caputo'. Several numerical approaches are available for the numerical approximation of such equations, yielding systems with (half-) full matrices. An alternative is to use a doubling-splitting approach to the operators in the PDE. Firstly, the operators are doubled to get a higher-order PDE, and then this PDE is split again into a system of lower-order PDEs, now giving a band-matrix structure. The method-of-lines procedure for approximating solutions of this new PDE system will be explained in more detail and illustrated with a series of numerical experiments. Analysis of the spectrum of the final system reveals that a special treatment of the time-integration is necessary to avoid numerical instabilities.