Symmetry of general linear methods and the underlying one-step method

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Let $\mathcal{M}_h : \mathbb{R}^r \to \mathbb{R}^r$ denote the map defined by

$$\begin{aligned} Y &= hAF + Uy, \\ F_i &= f(Y_i), \qquad i = 1, 2, \ldots, s, \\ \mathcal{M}_h y &= hBF + Vy. \end{aligned}$$

A method is "symmetric" if there exists an involution $L : \mathbb{R}^r \to \mathbb{R}^r$, $L^2 = I$, such that $\mathcal{M}_{-h} = L\mathcal{M}_h L$. Let Φ_h denote the underlying one-step method and $\mathcal{S}_h$ the corresponding starting method, so that $\mathcal{M}_h\mathcal{S}_h = \mathcal{S}_h\Phi_h$. This talk will include an analysis of the order of the method and related properties of $\mathcal{S}_h$ and Φ_h.