Convergence of AMF-Radau-type methods for the time integration of advection diffusion reaction PDEs

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A family of methods for the time integration of evolutionary Partial Differential Equations (PDEs) of Advection Diffusion Reaction type semi-discretized in space by Finite Differences is presented. These methods consider up to three inexact Newton Iterations of Approximate Matrix Factorization type (AMF) applied to the two-stage Radau IIA method along with a very simple predictor. The overall process reduces the algebraic costs involved in the numerical solution of the multidimensional linear systems to the $1D-$level.

Some specific AMF-Radau methods are selected after studying the expression for the local error in semi-linear equations, and their linear stability properties are described. The wedge of stability of the methods depends on the number of splittings used for the Jacobian matrix of the spatial semidiscretized ODEs, $J_h = \sum_{j=1}^{d} J_{h,d}$, where h stands for the spatial grid resolution. A-stability is obtained for the cases $d = 1, 2$, and $A(0)$-stability for any $d \geq 1$.

Numerical experiments on a semi-linear test problem with Dirichlet boundary conditions reveal that the AMF-Radau methods can attain order two (resp. three) in time just by giving one iteration (resp. two iterations) per integration step. A theory supporting the uniform convergence of order two and three on time, independently of the spatial resolution h, is presented. Uniform bounds for the global time-space errors when simultaneously the time step-size $\tau \to 0^+$ and the spatial grid resolution $h \to 0^+$ are obtained when the multidimensional PDEs are semi-linear with time-independent Dirichlet Boundary Conditions. For the case of time-dependent Boundary Conditions, a Boundary Correction Technique is proposed in order to avoid the order reduction phenomenon.