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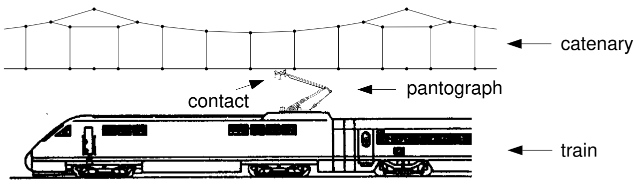
A pre-conditioned method for the dynamical simulation of coupled mechanical multibody systems

In the integrated design of complex technical systems the coupling of highly developed specialized simulation tools gets more and more important ("co-simulation"). The associated mathematical problem are coupled differential equations that have to be solved numerically. In the present paper we consider a waveform relaxation method for coupled mechanical multibody systems and use a pre-conditioner to guarantee convergence. The method has been applied successfully to the coupled simulation of pantograph and catenary.

1. Co-simulation

Traditionally, the components of complex technical systems have been designed and optimized more or less separately. Powerful specialized simulation tools have been developed for the computer aided model setup, for the automatic generation of the model equations, and for the post-processing of simulation results. Typically, these tools make use of very efficient numerical methods that are carefully tailored to the specific structure of the model equations.

The increasing computing power makes it nowadays possible to substitute more and more the traditional analysis of isolated system components by the integrated design of complex technical systems. A promising and frequently used approach to the simulation of such complex systems is based on the coupling of specialized simulation tools for the individual system components. During simulation the specialized tools run in parallel and exchange data at discrete time instants (*co-simulation*).



Example 1. *The system pantograph/catenary is the most critical part in the power supply of modern high-speed trains. New design concepts for the pantograph are considered to reduce wear and aeroacoustic noise. These investigations are essentially based on computer simulations [1],[4]. Veitl et al. [5] combine a mechanical multibody system (MBS) model for the pantograph with a beam model for the catenary and connect both models by the contact condition between pantograph head and contact wire. The simulation results are obtained by co-simulation coupling the industrial MBS package SIMPACK with the catenary simulation package PrOSA of Deutsche Bahn AG.*

2. A numerical method for coupled MBS models

Example 1 illustrates that the typical mathematical background of co-simulation is given by coupled differential equations that have to be solved numerically coupling the numerical methods of the individual specialized simulation tools. In the following we consider the dynamical simulation of two coupled MBS. The equations of motion are

$$\begin{aligned} M_i(q_i)\ddot{q}_i &= f_i(q_i,\dot{q}_i) - G_i^T(t,q_1,q_2)\lambda\ , \quad (i=1,2)\ , \\ 0 &= g(t,q_1,q_2) \end{aligned} \tag{1}$$

with the position coordinates q_1, q_2 of the first and second subsystem, respectively. The mass matrices M_i are supposed to be symmetric positive definite, the vectors f_i represent forces and momenta. Both subsystems are coupled by n_λ holonomic constraints $g = 0$ that result in constraint forces $-G_i^T\lambda$ with $G_i := \partial g/\partial q_i$ and $\lambda \in \mathrm{I\!R}^{n_\lambda}$. Eqs. (1) form a differential-algebraic system of index 3. Initial value problems for (1) are under reasonable assumptions uniquely solvable on a finite time interval $[0,T]$ ([2]). Example 1 results in equations of motion of the form (1) if the beam model is semi-discretized in space by finite elements or finite differences. In this application the subsystems are pantograph and catenary that are coupled by the scalar contact condition $g = 0$ [1].

For the numerical solution of (1) we introduce a discrete time grid $0 = T_0 < T_1 < \ldots < T_N = T$ on $[0,T]$ and continue stepwise from $t = 0$ to $t = T$. In step $T_n \to T_{n+1}$ the numerical solution is already known on $[0,T_n]$ and has to be continued on $[T_n,T_{n+1}]$. Generalizing waveform relaxation methods [3] to the index-3 system (1) we set

$$q_i\big|_{[T_n,T_{n+1}]} := q_{n,i}^{(k_n)}\ , \quad (i=1,2)\ , \qquad \lambda\big|_{[T_n,T_{n+1}]} := \lambda_n^{(k_n)}$$

with some given $k_n \geq 1$ and functions $q_{n,1}^{(k)}$, $q_{n,2}^{(k)}$, $\lambda_n^{(k)}$ that are defined iteratively by the initial value problems

$$M_1(q_{n,1}^{(k)})\ddot{q}_{n,1}^{(k)} = f_1(q_{n,1}^{(k)}, \dot{q}_{n,1}^{(k)}) - G_1^T(t, q_{n,1}^{(k)}, q_{n,2}^{(k-1)})\lambda_n^{(k-1)} \;, \quad q_{n,1}^{(k)}(T_n) = q_1(T_n) \;, \;\; \dot{q}_{n,1}^{(k)}(T_n) = \dot{q}_1(T_n) \tag{2}$$

and

$$\begin{aligned} M_2(q_{n,2}^{(k)})\ddot{q}_{n,2}^{(k)} &= f_2(q_{n,2}^{(k)}, \dot{q}_{n,2}^{(k)}) - G_2^T(t, q_{n,1}^{(k)}, q_{n,2}^{(k)})\Lambda_n^{(k)} \;, \quad q_{n,2}^{(k)}(T_n) = q_2(T_n) \;, \;\; \dot{q}_{n,2}^{(k)}(T_n) = \dot{q}_2(T_n) \\ 0 &= g(t, q_{n,1}^{(k)}, q_{n,2}^{(k)}) \;. \end{aligned} \tag{3}$$

The new iterate $\lambda_n^{(k)}$ is computed from $\Lambda_n^{(k)}$ using a pre-conditioning matrix $A_n \in \mathbb{R}^{n_\lambda \times n_\lambda}$ that has to be chosen such that $I - A_n$ is non-singular:

$$\Lambda_n^{(k)} = (I - A_n)\lambda_n^{(k)} + A_n \lambda_n^{(k-1)} \;. \tag{4}$$

The iteration starts with $k = 1$ and initial guesses $q_{n,2}^{(0)}$, $\lambda_n^{(0)}$ that are obtained extrapolating $q_2|_{[0,T_n]}$, $\lambda|_{[0,T_n]}$ to $[T_n, T_{n+1}]$. Then $q_{n,1}^{(1)}$ is computed as solution of (2). With this function $q_{n,1}^{(1)}$ we get $q_{n,2}^{(1)}$ and $\Lambda_n^{(1)}$ from (3) and $\lambda_n^{(1)}$ from (4) such that the iteration may be continued with $k = 2$ in (2) and so on. Method (2)–(4) is tailored to co-simulation since both subsystems are decoupled and may be solved separately by any standard method [2].

Eq. (4) would be simplified if $A_n := 0$, i. e., $\lambda_n^{(k)} = \Lambda_n^{(k)}$. But this choice of A_n results in exponential instability if the method is applied to Example 1 with the catenary as first and the pantograph as second subsystem. Extending results from circuit simulation [3] it may be shown that the *contractivity condition* $\alpha < 1$ with

$$\alpha := \sup_{t \in [0,T]} \| \, I - (I - A_n)^{-1} \, (\, I + (G_2 M_2^{-1} G_2^T)^{-1} \, (G_1 M_1^{-1} G_1^T) \,) \, \|$$

is necessary for convergence of the waveform relaxation method. (A more refined analysis shows that this condition is also sufficient for convergence, but this analysis is outside the scope of the present paper.) An optimal rate of convergence is achieved with the pre-conditioner $A_n = -(G_2 M_2^{-1} G_2^T)^{-1} \, (G_1 M_1^{-1} G_1^T)$ that results in $\alpha = 0$.

3. Application to the coupled dynamical simulation of pantograph and catenary

The waveform relaxation method has been tested for the benchmark problem "pantograph/catenary" of [1]. If the pantograph is chosen as first subsystem then there is no need for pre-conditioning since the contractivity condition is already satisfied with $\alpha < 0.04$ if $A_n := 0$. Using time steps $T_{n+1} - T_n$ in the range of $10^{-4} \ldots 10^{-2}$ s we found that typically $k_n = 1$ or $k_n = 2$ iterations in the waveform relaxation method give sufficiently accurate results.

In the decoupled numerical solution of (2) and (3) order and stepsize are adapted independently to the two subsystems. For stability reasons stepsizes $\Delta_t < 10^{-4}$ s are necessary in the subsystem catenary since the solution has nearly undamped oscillatory components. Up to 100 times larger stepsizes may be used for the pantograph such that the overall computing time is reduced by more than 70%. Similar savings have been obtained in the application to real technical systems using the waveform relaxation method in the co-simulation of SIMPACK and PrOSA.

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4. References

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