Bemerkung 4.10: Diskrete Fourier-Transformation

Trigonometrische Interpolation

$(N = 2n + 1)$

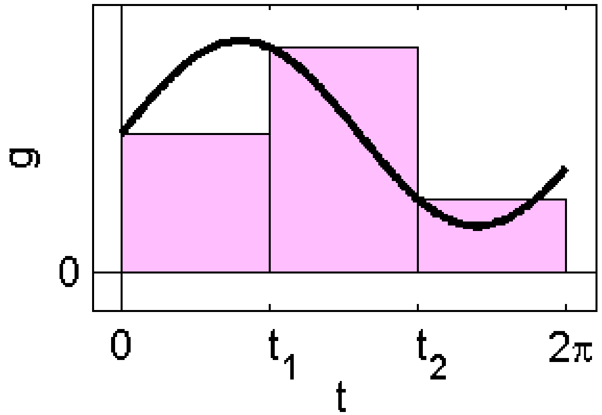
$$\phi_N(t_k) = \sum_{j=0}^{N-1} c_j \mathrm{e}^{ijt_k} = \sum_{j=-n}^{n} c_j \mathrm{e}^{ijt_k}$$

2π–periodische Funktionen $f \in L^2(\mathbb{R}) \;\Rightarrow\;$ (abgebrochene) Fourier–Reihe

$$f_n(t) := \sum_{j=-n}^{n} \hat{f}(j)\, \mathrm{e}^{ijt} \quad \text{mit } \underline{\text{Fourierkoeffizienten}} \quad \hat{f}(j) := \frac{1}{2\pi} \int_0^{2\pi} f(t) \mathrm{e}^{-ijt}\, \mathrm{d}t$$

Approximation

$$\int_0^{2\pi} g(t)\, \mathrm{d}t \;\approx\; \frac{2\pi}{N} \sum_{k=0}^{N-1} g(t_k)$$



$$\hat{f}(j) = \frac{1}{2\pi} \int_0^{2\pi} f(t) \mathrm{e}^{-ijt}\, \mathrm{d}t \;\approx\; \frac{1}{N} \sum_{k=0}^{N-1} f(t_k) \mathrm{e}^{-ijt_k} = \frac{1}{N} \sum_{k=0}^{N-1} f(t_k) \omega_k^{-j} = c_j$$