

Bemerkung 4.10: Diskrete Fourier-Transformation

Trigonometrische Interpolation

($N = 2n + 1$)

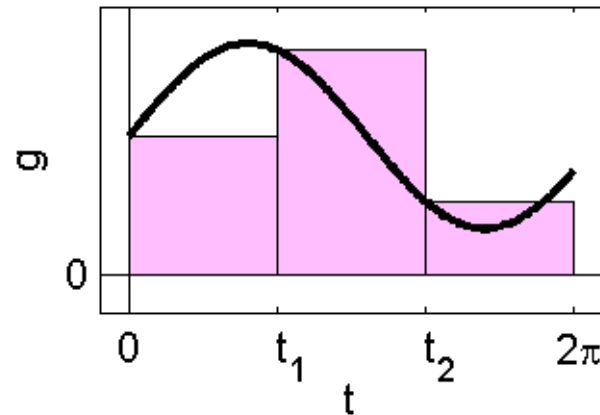
$$\phi_N(t_k) = \sum_{j=0}^{N-1} c_j e^{ijt_k} = \sum_{j=-n}^n c_j e^{ijt_k}$$

2π -periodische Funktionen $f \in L^2(\mathbb{R}) \Rightarrow$ (abgebrochene) Fourier-Reihe

$$f_n(t) := \sum_{j=-n}^n \hat{f}(j) e^{ijt} \quad \text{mit Fourierkoeffizienten } \hat{f}(j) := \frac{1}{2\pi} \int_0^{2\pi} f(t) e^{-ijt} dt$$

Approximation

$$\int_0^{2\pi} g(t) dt \approx \frac{2\pi}{N} \sum_{k=0}^{N-1} g(t_k)$$



$$\hat{f}(j) = \frac{1}{2\pi} \int_0^{2\pi} f(t) e^{-ijt} dt \approx \frac{1}{N} \sum_{k=0}^{N-1} f(t_k) e^{-ijt_k} = \frac{1}{N} \sum_{k=0}^{N-1} f(t_k) \omega_k^{-j} = c_j$$

