Bemerkung 1.12: Newton-Darstellung

Lagrangesche Basispolynome (vgl. Bemerkung 1.9)

$$L_j^{(n)}(x) := \prod_{\substack{i=0 \\ i \neq j}}^{n} \frac{x - x_i}{x_j - x_i} = \frac{(x - x_0)\,(x - x_1)\,\cdots\,(x - x_{i-1})\,(x - x_{i+1})\,\cdots\,(x - x_n)}{(x_j - x_0)(x_j - x_1)\cdots(x_j - x_{i-1})(x_j - x_{i+1})\cdots(x_j - x_n)}$$

Interpolationspolynom (rekursive Definition)

$$\Phi_{i,i+1,\ldots,i+l}(x) = \Phi_{i,i+1,\ldots,i+l-1}(x) + (x - x_i)(x - x_{i+1})\cdots(x - x_{i+l-1}) f_{i,i+1,\ldots,i+l}$$

Koeffizientenvergleich in

$$\Phi_{i,\ldots,i+l}(x) = \frac{(x - x_i)\Phi_{i+1,\ldots,i+l}(x) - (x - x_{i+l})\Phi_{i,\ldots,i+l-1}(x)}{x_{i+l} - x_i} \qquad (*)$$

$$\Rightarrow \qquad f_{i,i+1,\ldots,i+l} = \frac{f_{i+1,\ldots,i+l} - f_{i,\ldots,i+l-1}}{x_{i+l} - x_i}$$