

The Hankel norm for quadrature methods applied to random dynamical systems

Roland Pulch (Ernst-Moritz-Arndt-Universität Greifswald)

We consider linear dynamical systems in form of ordinary differential equations. The Hankel norm represents a bound for the ratio between the energy of outputs and inputs in the system. For modelling uncertainties, we replace physical parameters of the system by random variables. The random process, which satisfies the dynamical system, is expanded into a series with orthogonal basis functions. We define a Hankel norm for a truncated expansion, although it does not solve some dynamical system. The unknown coefficients of the truncated expansion represent probabilistic integrals. Thus they can be computed approximately by a quadrature method like sparse grid techniques or quasi Monte-Carlo schemes, for example. We arrange all involved linear dynamical systems into a much larger system, which is coupled by the outputs only. For the coupled linear system, the Hankel norm exists in the usual sense. We prove that these Hankel norms of the approximate systems converge to the Hankel norm associated to the exact truncated series provided that the quadrature method is convergent. Furthermore, numerical results of a test example are presented, where different quadrature techniques are involved.