Some families of W-methods to stabilize standard explicit Runge-Kutta methods for stiff problems

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A technique to stabilize standard explicit Runge-Kutta methods, by converting them in W-methods, is proposed. The main point to get the associated family of W-methods for a given explicit Runge-Kutta method is to use some smart simplification to reduce the large number of order conditions that must be sasitsfied for a W-method to reach a pre-fixed order (8 order conditions for order three, 21 conditions for order four, 58 conditions for order five, see e.g. Hairer and Wanner [1, Chap. IV.7]).
Based on this idea, for any given 4-stage explicit Runge-Kutta method, two uni-parametric families of third order W-methods are obtained. The free parameter (namely θ) can be used to increase the stability regions of the associated W-methods. In fact, it is possible to find L-stable ROW-methods (i.e., Rosenbrock-Wanner methods) in the family for some specific values of the parameter θ.
The new family of W-methods are also equiped with the splitting provided by the Approximate Matrix Factorization (AMF) [3], which converts the W-method in some kind of ADI-method (Alternating Direction Implicit method, see e.g. [2, Chap. IV]). This kind of methods is mainly used to solve large time-dependent PDE systems (2D or 3D spatial variables) discretized in space by using finite differences or finite volumes. For the case of d-splittings ($d = 1, 2, 3$), some stability properties of the family of W(AMF)-methods are analyzed.
Some numerical experiments, by applying the W(AMF)-methods, on a few interesting stiff problems coming from PDE discretizations are presented. The goal is to illustrate the relevance of the stability properties and the order reached by the methods and to see how they perform when compared with others proposed in the literature.

References

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[3] P.J. van der Houwen, B.P. Sommeijer, *Approximate factorization for time-dependent partial differential equations*, J. Comput. Appl. Math. 128 (2001), 447-466.