An Algorithm with Global Error Control for the Numerical Solution of the Generalized Density Profile Equation

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We are concerned with a generalization of the Cahn-Hilliard continuum model for multiphase fluids [1] where the classical Laplacian has been replaced by a degenerate one (i.e., so-called p-Laplacian). Using spherical symmetry, the model can be reduced to a boundary value problem for a second order nonlinear ordinary differential equation. One searches for a monotone solution of this equation which satisfies certain boundary conditions.
The case of the classical Laplacian was studied in detail in [2] and [3], while the generalized equation was analysed in [4] and [5]. It was proved that the arising boundary value problem possesses a unique increasing solution, under certain restrictions on the parameters. The asymptotic behavior of the solution has been analysed at two singular points; namely, at the origin and at infinity. From the point of view of numerical approximation, this problem presents some challenges, due to the singularities and to the existence of an interior layer where the derivative of the solution may take very high values. An efficient numerical technique for treating such singular boundary value problems has been presented, based on the shooting method and on nested implicit Runge-Kutta formulas with global error control. The present talk will focus mainly on the latter approach. We will briefly describe the computational algorithm and present some numerical results, which are highly accurate and in agreement with the experimental ones.

References

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