Trigonometrically fitted numerical methods for reaction-diffusion problems

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It is the purpose of this talk to present novel finite difference schemes for reaction-diffusion problems, which generate traveling waves as fundamental solutions. Such problems have been widely employed as mathematical model for life science phenomena exhibiting the generation of periodic waves along their dynamics (e.g. cell cycles, frequently behaving if they are driven by an autonomous biochemical oscillator; intracellular calcium signalling, since calcium shows many differrent types of oscillations in time and space, in response to various extracellular signals).

The periodic character of the problem suggests to propose a numerical solution which takes into account this behavior, i.e. by tuning the numerical solver to accurately and efficiently follow the oscillations appearing in the solution, since classical numerical methods would require the employ of a very small stepsize to accurately reproduce the dynamics.

For this reason, we propose an adaptation of classical finite difference schemes which will take into account the qualitative nature of the solutions, in a problem-oriented setting. We may say that a three-fold level of adaptation to problem will be carried out: along time and space, by suitable semidiscretization with problem-based finite differences and analog time solvers for the semi-discrete problem, and along the problem by taking into account the peculiarity of the vector field, for instance by proposing suitable novel implicit-explicit schemes.

The corresponding numerical method will depend on variable coefficients, which are functions of the parameters characterizing the solution (e.g. the frequency of the oscillations). A theoretical study of the error associated to the overall numerical scheme gives us the possibility to propose an accurate estimate of the unknown parameters on which the numerical method depends.

Practical constructive aspects and accuracy analysis will be treated, as well as numerical experiments showing the effectiveness of the approach will be provided.

References

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