Numerical methods in the simulation of vehicle-guideway interaction

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Summary. Trains and heavy trucks crossing bridges result in loads that may cause wear and damage. The damages are substantially reduced by an optimal design of bridges and vehicle suspensions. To improve the technical construction the dynamical interaction between vehicles and their guideways is analysed by computer simulations. The coupled problem vehicle–guideway is studied coupling industrial simulation packages and numerical methods from multibody dynamics (vehicle) and structural mechanics (elastic structure). In the present paper numerical aspects of this modular approach are discussed and an adapted modular time integration scheme is introduced. The methods have been used successfully in practical applications.

1 Introduction

The theoretical analysis of vehicle–guideway interaction goes back to the early days of railway engineering in the 19th century, see Section 1.3 of [Lut02] for a historical overview. Traditionally, the investigations focussed on one of the two subsystems and the influence of the complementary second subsystem was considered by strongly simplified models only.

In structural mechanics the main interest is in bridges and guideways. The influence of tyre forces and wheel-rail contact forces and the vehicle's eigendynamics are considered by moving loads on the elastic structure and by harmonic oscillators [Frý96]. Vehicle–guideway interaction is investigated as well in vehicle dynamics with detailed multibody vehicle models subject to excitations reflecting the dynamical response of the elastic structure [KL94].

These classical approaches allow an efficient qualitative analysis of interaction phenomena. However, for a reliable quantitative analysis of a given technical system more precise models for vehicle and guideway are necessary. In industrial applications these models are typically set up in state-of-the-art simulation packages but there is no commercial simulation tool that supports the model setup both for the vehicle and for the guideway.

Lutzenberger [Lut02] extends the industrial finite element (FE) package NASTRAN by a method for the coupled time integration of a standard FE bridge model and (linear) truck models consisting of beam elements. Duffek [Duf91, DS01] uses the multibody system (MBS) packages MEDYNA and SIMPACK and develops a user force element that computes interaction forces and momenta for vehicles moving on a bridge modeled as beam structure.

Instead of extending one given simulation tool we study in the present paper the *coupling* of two different tools for simulating vehicle–guideway interaction. In this *co-simulation* approach vehicle and guideway are modeled separately in MBS package and FE package, respectively, see Sections 2 and 3. The coupled system is integrated in time by a modular method, see Section 4.

The present work was part of a joint project of DLR, INTEC GmbH Wessling and the Chair of Structural Mechanics at Munich University of Technology (Prof. Dr.-Ing. H. Grundmann) on vehicle–guideway interaction, see [DHS01] for a discussion of practical implementation and industrial applications. The cooperation with all project partners is gratefully acknowledged. The author would like to thank especially Dr.-Ing. S. Dietz (INTEC GmbH) for providing the simulation results that are shown in Section 5.

2 Simulation of vehicle dynamics

The analysis of vehicle dynamics is based on multibody system models that consist of a finite number of rigid or elastic bodies and the joints and force elements that connect these bodies [KL94]. The equations of motion are

$$M_{\mathrm{V}}(q_{\mathrm{V}})\,\ddot{q}_{\mathrm{V}}(t) = f_{\mathrm{V}}(t, q_{\mathrm{V}}, \dot{q}_{\mathrm{V}}, q_{\mathrm{b}}, \dot{q}_{\mathrm{b}}, \lambda_{\mathrm{V}}) - G_{\mathrm{V}}^{\top}(t, q_{\mathrm{V}}, q_{\mathrm{b}})\,\lambda_{\mathrm{V}}\,, \tag{1}$$

$$0 = g_{\mathrm{V}}(t, q_{\mathrm{V}}, q_{\mathrm{b}}) \tag{2}$$

with q_{V} denoting the position coordinates of all bodies in the **v**ehicle model and q_{b} describing the elastic deformation of the **b**ridge. $M_{\mathrm{V}}(q_{\mathrm{V}})$ is the generalized mass matrix and f_{V} the vector of applied forces. Joints decrease the number of degrees of freedom in the system and may result in constraints (2) that are coupled to the dynamical equations (1) by constraint forces $-G_{\mathrm{V}}^{\top}\lambda_{\mathrm{V}}$ with the Lagrange multipliers λ_{V} and the constraint matrix $G_{\mathrm{V}}(t,q) := (\partial g_{\mathrm{V}}/\partial q_{\mathrm{V}})(t, q_{\mathrm{V}})$.

In (1)/(2) the interaction of the vehicle with the bridge is considered by tyre forces f_{V} between a truck and its guideway or by the wheel-rail contact that results in constraints (2) and corresponding constraint forces $-G_{\mathrm{V}}^{\top}\lambda_{\mathrm{V}}$.

For the time integration of (1)/(2) the second order differential-algebraic equation is rewritten as first order system and variable stepsize variable order BDF are applied to its *stabilized index-2 formulation* that was first considered by Gear, Gupta and Leimkuhler [BCP96]. Industrial MBS simulation packages like ADAMS and SIMPACK use specially adapted versions of the DASSL time integration software [BCP96].

Even for complex MBS models the simulation time on PC hardware is often close to real time. Simulations, parameter variations and the optimization of system parameters may be performed in reasonable computing time.

Full vehicle models consist of up to 50 bodies resulting in more than 100 state variables, see Fig. 1 for an example. Typical computing times for a guideway of length 100 m are in the range of 2 s for a truck model and in the range of 150 s for the substantially more complex and nonlinear model of a rail vehicle (SIMPACK model on a Pentium III PC with 500 MHz).

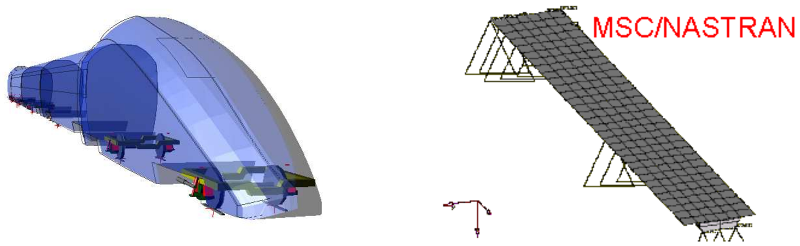

Fig. 1. Left: MBS model of an articulated body train in SIMPACK. Right: FE model of a road bridge in NASTRAN.

3 Dynamical simulation of road and railway bridges

Industrial finite element packages like NASTRAN are used for the modeling of the bridge, see Fig. 1. The resulting equations of motion have the form

$$M_{\mathrm{b}}\,\ddot{w} + D_{\mathrm{b}}\,\dot{w} + K_{\mathrm{b}}\,w = p_{\mathrm{b}}(F(t, q_{\mathrm{v}}, \dot{q}_{\mathrm{v}}, \lambda_{\mathrm{v}}))\,. \tag{3}$$

Here M_{b}, D_{b}, K_{b} denote mass, damping and stiffness matrix of the **b**ridge model. The load vector p_{b} is determined by the forces $F(t, q_{\mathrm{v}}, \dot{q}_{\mathrm{v}}, \lambda_{\mathrm{v}})$ that depend on the state $(q_{\mathrm{v}}, \dot{q}_{\mathrm{v}}, \lambda_{\mathrm{v}})$ of the vehicle model and represent tyre forces or wheel-rail contact forces.

In principle the MBS model (1)/(2) could be coupled directly to this finite element model (3) with $n_w \approx 10^4$ degrees of freedom. But modal reduction with $n_{\mathrm{b}} \approx 100$ eigenmodes φ_j gives nearly identical results and reduces the numerical effort drastically [DHS01]. After modal reduction the equations of motion may be decoupled into the system

$$m_j\,\ddot{q}_{\mathrm{b},j} + d_j\,\dot{q}_{\mathrm{b},j} + k_j\,q_{\mathrm{b},j} = \varphi_j^{\top} p_{\mathrm{b}}(F(t, q_{\mathrm{v}}, \dot{q}_{\mathrm{v}}, \lambda_{\mathrm{v}}))\,,\ \ (j = 1, \ldots, n_{\mathrm{b}})\,. \tag{4}$$

In the co-simulation framework of Section 4 Eqs. (4) have to be solved on time intervals $[T_n, T_{n+1}]$ of length ≤ 1 ms. Therefore the force terms $\varphi_j^{\top} p_{\mathrm{b}}$ on the right hand sides of (4) may be approximated by *linear* right hand sides $\bar{p}_{j,o}^{(n)} + \dot{\bar{p}}_{j,o}^{(n)} \cdot (t - T_n)$ without significant loss of accuracy.

The constant coefficients $\bar{p}_{j,o}^{(n)}$ and $\dot{\bar{p}}_{j,o}^{(n)}$ are determined by T_n, $q_{\mathrm{v}}(T_n)$, $\dot{q}_{\mathrm{v}}(T_n)$, $\ddot{q}_{\mathrm{v}}(T_n)$ and $\lambda(T_n)$. The resulting n_{b} scalar linear second order differential equations are solved analytically. With this combination of modal reduction techniques and semi-analytical methods the numerical solution of (3) may be obtained very efficiently.

In the practical implementation a technical problem remains to be solved: The eigenmodes φ_j are exported pointwise from the FE package but the vehicle moves continuously on the bridge's surface. Therefore the pointwise given φ_j are interpolated by continuously differentiable cubic tensorproduct splines that are defined locally based on local data. These Akima like splines may be computed and evaluated very efficiently and give a smooth, non-oscillating interpolation of the huge set of data points [Aki70].

4 Co-Simulation of the coupled system vehicle–guideway

The modular structure of the coupled system vehicle–guideway may not only be exploited in the model setup (see Sections 2 and 3) but also in the time integration of the coupled system (1)/(2)/(4) using for each subsystem its own time integration method (*co-simulation*). The communication between the two subsystems is restricted to discrete synchronization points T_n.

In the beginning of each *macro step* $[T_n, T_{n+1}]$ the forces $F(T_n, q_{\rm V}(T_n), \dot{q}_{\rm V}(T_n), \lambda_{\rm V}(T_n))$ are transferred to the elastic structure. In Stage 1 of the macro step the loads $p_{\rm b}(F)$ are used to determine the coefficients $\bar{p}_{j,o}^{(n)}$ and $\dot{\bar{p}}_{j,o}^{(n)}$ and to solve (4). The resulting elastic deformation $q_{\rm b}(t)$ of the bridge is added to the track irregularities in the MBS tool that influence the tyre forces $f_{\rm V}$ (truck) or the wheel-rail contact conditions (2) and the contact forces $-G_{\rm V}^{\top}\lambda_{\rm V}$ (rail vehicle). In Stage 2 of the macro step the equations of motion (1)/(2) for the vehicle are integrated for $t \in [T_n, T_{n+1}]$ using standard methods from multibody dynamics like BDF.

This co-simulation technique is convenient but it is well known from the literature that it may suffer from numerical instability. For certain classes of coupled problems the instability phenomenon has been analysed in great detail. Several modifications of the co-simulation techniques help to improve its stability, accuracy and robustness [KS00, AG01].

In the application to vehicle–guideway interaction it is important to start in Stage 1 with the elastic structure and not with the vehicle. Practical experience shows that the algorithm is stable and sufficiently accurate if the macro stepsize $H := T_{n+1} - T_n$ is in the range of 1.0 ms.

5 Practical application

Dietz et al. [DHS01] study as a typical application two trains that pass each other on a bridge. Fig. 2 shows the vertical displacement of the bridge at the fixed position $x_b = 28.0\,\mathrm{m}$. The first train reaches x_b after 2.63 s and the second train after 7.06 s. At both time instances the elastic deformation reaches a local maximum.

In the simulation a Pentium III PC was used with sophisticated nonlinear multibody system models for both trains ($n_{\rm V} > 100$). The coupled simulation

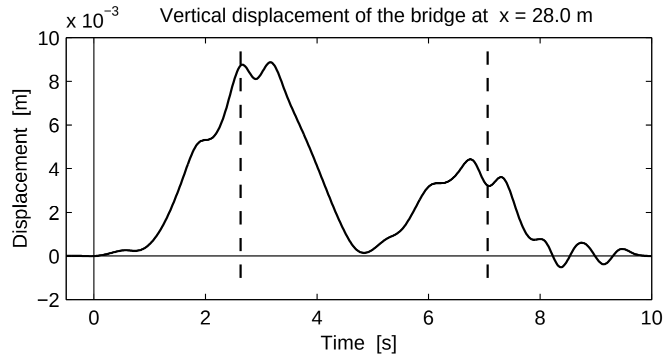

Fig. 2. Vertical displacement of a bridge with two trains passing each other.

was performed in 580 s cpu-time compared with a cpu-time of 295 s for the pure multibody simulation of the two vehicles. The co-simulation approach allows an efficient dynamical simulation of this complex engineering problem.

References

[AG01] M. Arnold and M. Günther. Preconditioned dynamic iteration for coupled differential-algebraic systems. *BIT Numerical Mathematics*, 41:1–25, 2001.

[Aki70] H. Akima. A new method of interpolation and smooth curve fitting based on local procedures. *Journal of the ACM*, 17:589–602, 1970.

[BCP96] K.E. Brenan, S.L. Campbell, and L.R. Petzold. *Numerical solution of initial–value problems in differential–algebraic equations.* SIAM, Philadelphia, 2nd edition, 1996.

[DHS01] S. Dietz, G. Hippmann, and G. Schupp. Interaction of vehicles and flexible tracks by co–simulation of multibody vehicle systems and finite element track models. 17th IAVSD Symposium, August 20–24, Copenhagen, 2001.

[DS01] W. Duffek and G. Schupp. The guideway operator as element of SIMPACK for the simulation of vehicle–guideway interaction, (in German). In M. Arnold, editor, *Workshop "Fahrzeug–Fahrweg–Wechselwirkung", Oberpfaffenhofen*, pages 29–35, DLR, Technical Report IB 532–01–04, 2001.

[Duf91] W. Duffek. Ein Fahrbahnmodell zur Simulation der dynamischen Wechselwirkung zwischen Fahrzeug und Fahrweg. Technical Report IB 515–91–18, DLR, D-5000 Köln 90, 1991.

[Frý96] L. Frýba. *Dynamics of Railway Bridges.* Thomas Telford, London, 1996.

[KL94] W. Kortüm and P. Lugner. *Systemdynamik und Regelung von Fahrzeugen.* Springer–Verlag, Berlin Heidelberg New York, 1994.

[KS00] R. Kübler and W. Schiehlen. Two methods of simulator coupling. *Mathematical and Computer Modelling of Dynamical Systems*, 6:93–113, 2000.

[Lut02] S.H. Lutzenberger. *Ein differentiell–algebraisches Simulationsmodell zur Fahrzeug–Fahrweg–Interaktion.* PhD thesis, Munich University of Technology, Department of Civil Engineering and Geodesy, 2002.